The principle used is to introduce a deliberate and accurately measurable initial change of effective area — by varying the diameter of one of the components of the assembly — which is made to serve as a reference quantity in terms of which the additional changes of effective area due to pressure may be calculated from measurements of other quantities which vary with the applied pressure.

The procedure used is actually only one of a class of possible methods, of which others will be mentioned below. In the form adopted the rates of flow of the pressure-transmitting fluid through the interspace between the piston and cylinder are measured, at a series of applied pressures, using two alternative pistons having an accurately known difference of daimeter. A simple relation may then be developed connecting the changes of effective area due to distortion with the initial change due to the different piston diameter, and the rates of flow corresponding to the two pistons.

Two other methods of the same general nature, but not depending on flow measurement, were considered and some preliminary experiments carried out. In the first case the quantity measured was the rate of retardation of the rotation speed of the piston and loading weights due to fluid friction in the clearance between piston and cylinder, corresponding to the two piston diameters. It was found, however, that the contribution due to air friction on the rotating load system was an important factor, and rather elaborate measures would have been necessary to eliminate this effect. In the second case the intention was to compare the electrical capacitances of the piston-cylinder assembly corresponding to the two piston diameters. This method, on which so far only very preliminary trials have been made, would very likely repay further exploration, but a knowledge of the pressure dependence of the dielectric constant of the transmitting fluid would be required to complete the reduction of the experimental data.

b) Theory of the flow method

The main problem in the theory of the method is to establish a reasonably simple connection between the measured rates of flow of the pressure transmitting fluid and the corresponding changes of effective area at the same applied pressures.

To introduce the variation of effective area with pressure we adopt the formal expression (2.5) of section 2 b, in which the only term dependent upon h is the integral $\frac{2}{rP}\int_0^P h\,dp$. The remaining variable term, $P(3\sigma - 1)/E$, is a small part of the total, and it has already been seen that the assumption on which the derivation of this term is based is unlikely to lead to appreciable error.

Denoting by Q the volume velocity of the fluid through any section of the annular gap, and $\eta(x)$ the coefficient of viscosity of the fluid at the axial distance x, it is easily shown that, under conditions of viscous flow,

$$\frac{3Q}{4\pi r} = -\frac{dp}{dx}\frac{h^3}{\eta} \; * \qquad (5.1)$$

* To avoid unnecessarily complicating the notation we ignore variations of the density of the fluid with pressure, as these are very unimportant compared with the variations in the coefficient of viscosity.

and by direct integration, we have

$$\frac{3Q}{4\pi r} = \int_0^P \frac{h^3}{\eta}\,dp\,. \qquad (5.2)$$

In order to exhibit the direct relation between Q and $\int_0^P h\,dp$ in a suitable form we may integrate equation (5.1) by a different route, whence we obtain

$$\left(\frac{3Q}{4\pi r}\right)^{\frac{1}{3}} = -\int_0^P h\,dp \Bigg/ \int_0^P \left(\eta\frac{dx}{dp}\right)^{\frac{1}{3}} dp\,. \qquad (5.3)$$

This equation shows that the factor relating $Q^{\frac{1}{3}}$ to $\int_0^P h\,dp$ is a function only of the pressure distribution in the interspace between piston and cylinder, and is not explicitly dependent on h. This suggests the possibility that $\int_0^P \left(\eta\frac{dx}{dp}\right)^{\frac{1}{3}} dp$ may not vary very much for a moderate change in the initial diameter of the piston.

Re-arranging equations (5.2) and (5.3), and writing for brevity

$$\chi = \left(\frac{3Q}{4\pi r}\right)^{\frac{1}{3}} \quad \text{and} \quad I = -\int_0^P \left(\eta\frac{dx}{dp}\right)^{\frac{1}{3}} dp\,,$$

we have

$$\int_0^P h\,dp = \chi I\,; \; I = l^{\frac{1}{3}}\int_0^P h\,dp \Big/ \left(\int_0^P \frac{h^3}{\eta}\,dp\right)^{\frac{1}{3}}. \qquad (5.4)$$

The second of these equations provides the basis for the calculation of the integral factor I, connecting the required changes of effective area with the experimentally determined rates of flow.

Before considering further the evaluation of the integral I, it is convenient to convert the formal equations connecting the changes of effective area with the quantities χ and I to a form suitable for application to the experimental data. Proceeding from equation (2.5) and using suffixes 1, 2 where necessary to distinguish the two piston diameters, and denoting by δr the value of $(r_1 - r_2)$ we have

$$A_{P,1} = \pi r_1^2\left[1 + \frac{P}{E}(3\sigma - 1) + \frac{2}{r_1 P}\int_0^P h_1\,dp_1\right],$$

$$A_{P,2} = \pi r_2^2\left[1 + \frac{P}{E}(3\sigma - 1) + \frac{2}{r_2 P}\int_0^P h_2\,dp_2\right]$$

whence, ignoring terms of the second order of small quantities,

$$A_{P,1} + A_{P,2} = 2\pi r_1^2\left[1 + \frac{P}{E}(3\sigma - 1) - \frac{\delta r}{r} + \frac{1}{rP}\left(\int_0^P h_1\,dp_1 + \int_0^P h_2\,dp_2\right)\right],$$

and

$$A_{P,1} - A_{P,2} = 2\pi r_1^2\left[\frac{\delta r}{r} + \frac{1}{rP}\left(\int_0^P h_1\,dp_1 - \int_0^P h_2\,dp_2\right)\right].$$